Q1

1

Complete the table.

Degrees	Radians	sin	cos	tan
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
315°	$\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1

[3]

$180° = \pi$ radians, so

$$(\text{degrees}) \times \frac{\pi}{180} = (\text{radians})$$

$$(\text{radians}) \times \frac{180}{\pi} = (\text{degrees})$$

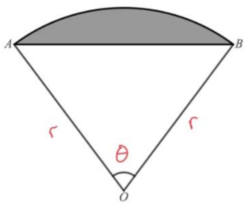
And don't forget,

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

Q2

2

The canopy of a parachute and the outermost suspension lines (cords) form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O.



The area of the sector OAB is $\frac{125\pi}{144}$ m^2.

The length of the arc AB is $\frac{25\pi}{36}$ m.

Find the length of one suspension line and the angle AOB that the parachutist makes with the two outermost suspension lines.

[5]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

arc length : $r\theta = \frac{25\pi}{36}$ ①

area : $\frac{1}{2}r^2\theta = \frac{125\pi}{144} \Rightarrow r^2\theta = \frac{125\pi}{72}$ ②

② ÷ ① : $r = \frac{125\cancel{\pi}/72}{25\cancel{\pi}/36} = \frac{125 \times 36}{25 \times 72}$

$$\boxed{r = \frac{5}{2} \text{ m (or } 2.5 \text{ m)}}$$

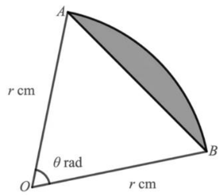
From ① : $\theta = \frac{25\pi}{36r} = \frac{25\pi}{36\left(\frac{5}{2}\right)}$

$$\boxed{\theta = \frac{5\pi}{18} \text{ radians}}$$

Q3a

3a

The diagram below shows the sector of a circle OAB.



(a) Show that the area of the shaded segment is given by $\frac{1}{2}r^2(\theta - \sin\theta)$ cm^2.

[3]

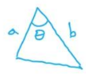
(b) Find, in terms of θ, the percentage of the sector that the segment occupies.

[2]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!



Area $= \frac{1}{2}ab\sin\theta$

a)

$$\text{Area of shaded segment} = \left(\text{Area of sector } OAB\right) - \left(\text{Area of triangle } OAB\right)$$

$$= \frac{1}{2}r^2\theta - \frac{1}{2}(r)(r)\sin\theta$$

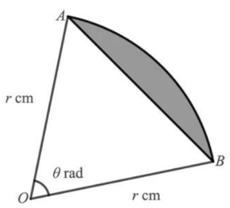
$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

Q3b

3b

The diagram below shows the sector of a circle OAB.



(a) Show that the area of the shaded segment is given by $\frac{1}{2}r^2(\theta - \sin\theta)$ cm^2.

[3]

(b) Find, in terms of θ, the percentage of the sector that the segment occupies.

[2]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

b)

$$\frac{\cancel{\frac{1}{2}r^2}(\theta - \sin\theta)}{\cancel{\frac{1}{2}r^2}\theta} \times 100$$

$$= \frac{\theta - \sin\theta}{\theta} \times 100$$

$$= 100\left(\frac{\theta - \sin\theta}{\theta}\right)$$

$$\boxed{= 100\left(1 - \frac{\sin\theta}{\theta}\right)}$$

Q4

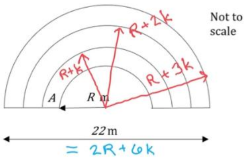
4

An evil wizard has captured a unicorn, and is threatening to kill it unless you can answer the following question:

"I wish to create a rainbow-shaped mosaic for the floor of the throne room in my castle. The mosaic is to be formed from four semicircles as shown below.

The innermost semicircle is to have a radius of R metres, and each of the outer semicircles must have a radius that is a constant k metres greater than the radius of the next semicircle further in.

The rainbow mosaic must be exactly 22 metres across.



Moreover, I wish the inner part of the area (labelled A in the diagram) to take up exactly one quarter of the total area of the rainbow.

Find me the required values of R and k, or the unicorn dies. Bwah-ha-ha-ha-ha!"

Solve the wizard's problem and save the unicorn.

[7]

area of a semicircle $= \frac{1}{2}\pi r^2$

area of entire rainbow $= \frac{1}{2}\pi(R+3k)^2 - \frac{1}{2}\pi R^2$

area of $A = \frac{1}{2}\pi(R+k)^2 - \frac{1}{2}\pi R^2$

$$\frac{\cancel{\tfrac{1}{2}\pi}(R+k)^2 - \cancel{\tfrac{1}{2}\pi}R^2}{\cancel{\tfrac{1}{2}\pi}(R+3k)^2 - \cancel{\tfrac{1}{2}\pi}R^2} = \frac{2\cancel{k}R + k^{\cancel{2}}}{6\cancel{k}R + 9k^{\cancel{2}}} = \frac{1}{4}$$

$\Rightarrow 8R + 4k = 6R + 9k \Rightarrow 2R = 5k$ ①

Also $2R + 6k = 22$ ②

Substitute ① into ② : $5k + 6k = 22 \Rightarrow k = 2$

Substitute $k=2$ into ① : $2R = 5(2) = 10 \Rightarrow R = 5$

$$\boxed{\text{The required values are } R = 5\text{ m} \quad k = 2\text{ m}}$$

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

} θ must be in radians!

← And note : A semicircle is a sector of a circle where the angle at the centre, θ, is π radians (180°)

Q5a

5a

A sector of a circle, OST, is such that it has radius r cm and the angle at its centre, O, is θ radians. The chord ST has length a cm.

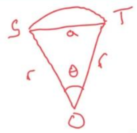
(a) Show that $a^2 = 2r^2(1 - \cos\theta)$

[2]

(b) Given that $r = 4\theta$ and that the area of the sector is $\frac{8\pi^3}{27}$ cm^2, find the value of a.

[5]

a)



By the cosine rule applied to triangle OST :

$a^2 = (r)^2 + (r)^2 - 2(r)(r)\cos\theta$

$a^2 = 2r^2 - 2r^2\cos\theta$

$a^2 = 2r^2(1 - \cos\theta)$

Q5b

5b

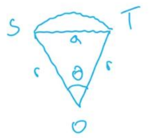
A sector of a circle, OST, is such that it has radius r cm and the angle at its centre, O, is θ radians. The chord ST has length a cm.

(a) Show that $a^2 = 2r^2(1 - \cos\theta)$

[2]

(b) Given that $r = 4\theta$ and that the area of the sector is $\frac{8\pi^3}{27}$ cm^2, find the value of a.

[5]



Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

b) $\frac{1}{2}r^2\theta = \frac{1}{2}(4\theta)^2\theta = \frac{8\pi^3}{27}$

$\Rightarrow \cancel{8}\theta^3 = \frac{\cancel{8}\pi^3}{27} \Rightarrow \theta^3 = \frac{\pi^3}{27} \Rightarrow \theta = \frac{\pi}{3}$

$a^2 = 2r^2(1-\cos\theta) = 2(4\theta)^2(1-\cos\theta)$

$\Rightarrow a^2 = 32\theta^2(1-\cos\theta)$

$a^2 = 32\left(\frac{\pi}{3}\right)^2\left(1-\frac{1}{2}\right)$

$a^2 = 32\left(\frac{\pi^2}{9}\right)\left(\frac{1}{2}\right)$

$a^2 = \frac{16\pi^2}{9}$

$a = \pm\frac{4\pi}{3}$

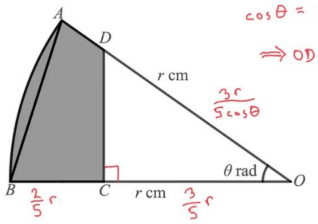
But a is a chord length, so it can't be negative

$$\boxed{a = \frac{4\pi}{3}}$$

Q6

6

The diagram below shows the sector of a circle with centre O. The radii OA and OB are each equal to r cm, and the angle at the centre, AOB, is equal to θ radians. The line DC is perpendicular to the line OB.

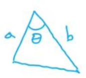


$\cos\theta = \frac{OC}{OD} = \frac{\frac{3}{5}r}{OD}$

$\Rightarrow OD = \frac{3r}{5\cos\theta}$

Given that $BC : CO = 2 : 3$, show that the area of the shaded shape $ABCD$ is given by $\frac{1}{50}r^2(25\theta - 9\tan\theta)$ cm^2

[6]



Area $= \frac{1}{2}ab\sin\theta$

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

area of shaded shape ABCD $=$ (area of sector AOB) $-$ (area of triangle OCD)

$= \frac{1}{2}r^2\theta - \frac{1}{2}\left(\frac{3}{5}r\right)\left(\frac{3r}{5\cos\theta}\right)\sin\theta$

$= \frac{1}{2}r^2\theta - \frac{1}{2}\left(\frac{9r^2}{25\cos\theta}\right)\sin\theta$

$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\left(\frac{9}{25}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1}{2}r^2\theta - \frac{9}{50}r^2\tan\theta$

$= \frac{1}{50}r^2(25\theta - 9\tan\theta)$ cm^2